Charged Null Fluid Collapse in Anti-de Sitter Spacetimes and Naked Singularities

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Abstract

We investigate the occurrence of naked singularities in the spherically symmetric, plane symmetric and cylindrically symmetric collapse of charged null fluid in an anti-de Sitter background. The naked singularities are found to be strong in Tipler's sense and thus violate the cosmic censorship conjecture, but not hoop conjecture.

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That the end state of gravitational collapse of a sufficiently massive star is a gravitational singularity is a fact established by the singularity theorems of Hawking and Ellis [1]. However, these singularities theorems does not guarantee the existence of an event horizon. The conjecture that such a singularity from a regular initial surface must always be hidden behind an event horizon, called cosmic censorship conjecture (CCC) was proposed by Penrose [2]. The CCC forbids the existence of naked singularities. Despite almost 30 years of effort we are far from a general proof of CCC (for recent reviews and references, see [3]). But, significant progress has been made in trying to find counter examples to CCC. In particular, the Vaidya [4] solution that represent an imploding null fluid with spherical symmetry has been intensively studied for the formation of naked singularities [5].

Many of studies in the gravitational collapse were motivated by Thorne's is hoop conjecture [6] that collapse will yield a black hole only if a mass M is compressed to a region with circumference $C \leq 4\pi M$ in all directions. If hoop conjecture is true, naked singularities may form if collapse can yield $C \geq 4\pi M$ in some direction. Thus, planar or cylindrical matter will not form a black hole (black plane or black string) [6].

However hoop conjecture was given for spacetimes with a zero cosmological term. In the presence of negative cosmological term one can expect the occurrence of major changes.

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When a negative cosmological constant is introduced, the spacetime will become asymptotically anti-de Sitter spacetime. Indeed, Lemos [7] has shown that planar or cylindrical black holes form rather than naked singularity from gravitational collapse of a planar or cylindrical matter distribution in an anti-de Sitter spacetime, violating in this way the hoop conjecture but not CCC. He also pointed out that for the spherical case, the collapse proceed to form naked singularities violating cosmic censorship conjecture, but not the hoop conjecture.

The purpose of this brief report is to see how the results found in ref. [7] get modified for the charged case. The usefulness of these models is that they do offer opportunity to explore of properties of singular spacetime and, in the case of curvature singularity to address issue such as local or global nakedness [8] and strength. Such a model may be valuable in attempts to put CCC in concrete mathematical form.

We find that both spherical and non-spherical collapse of charged null fluid admit strong curvature naked singularities in accordance with hoop conjecture and violating CCC.

The charged Vaidya anti-de Sitter metric in (v, r, θ, ϕ) coordinates is [9, 10]

$$ds^2 = -(1 + \alpha^2 r^2 - \frac{2m(v)}{r} + \frac{e^2(v)}{r^2})dv^2 + 2dvdr + r^2(d\theta^2 + sin^2\theta d\phi^2), \tag{1}$$

where $\alpha \equiv \sqrt{-\Lambda/3}$, Λ is the cosmological constant. v represents advanced Edmonton time, in which r is decreasing towards the future along a ray $v = const.$ and the two arbitrary functions $m(v)$ and $e(v)$ (which are restricted only by the energy conditions), represent, respectively, the mass and electric charge at advanced time v. This metric (1) represents a solution to the Einstein-Maxwell equations for a collapsing charged null fluid in the spherically symmetric anti-de Sitter background.

The model considered here is obtained from energy momentum tensor of the form

$$T_{ab} = \rho k_a k_b + T^{(m)}_{ab}, \tag{2}$$

where ρ in this case is given by

$$\rho = \frac{1}{4\pi r^3}\left[r\dot{m}(v) - e(v)\dot{e}(v)\right] \tag{3}$$

with the null vector k_a satisfying $k_a = -\delta^v_a \; and \; k_a k^a = 0$, $T^{(m)}_{ab}$ is related to the electromagnetic tensor F_{ab}:

$$T^{(m)}_{ab} = \frac{1}{4\pi}\left(F_{ac}F^c_b - \frac{1}{4}g_{ab}F_{cd}F^{cd}\right) \tag{4}$$

which satisfies Maxwell's field equations

$$F_{[ab;c]} = 0 \, and \, F_{ab;c}g^{bc} = 4\pi J_a, \tag{5}$$

where J_a is the four-current vector.

Clearly, for the weak energy condition to be satisfied we require the bracketed quantity in Eq. (3) to be non negative. We note that the stress tensor in general may not obey the weak energy condition. In particular, if $dm/de > 0$ then there always exists a critical radius $r_c = e\dot{e}/\dot{m}$ such that when $r < r_c$ the weak energy condition is always violated. However, in realistic situations, the particle cannot get into the region $r < r_c$ because of the Lorentz force and so the energy condition is still preserved [10, 11].

The Kretschmann scalar ($K = R_{abcd}R^{abcd}$, R_{abcd} is the Riemann tensor) for the metric (1) reduces to

$$K = \frac{48}{r^6}\left[m^2(v) - \frac{2}{r}e^2(v)m(v) + \frac{7}{6}\frac{e^4(v)}{r^2}\right] + 24\alpha^4. \tag{6}$$

So the Kretschmann scalar diverges along $r = 0$ for m and $e \neq 0$, establishing that metric (1) is scalar polynomial singular [1].

The physical situation is that of a radial influx of charged null fluid in the region of the anti-de Sitter universe. The first shell arrives at $r = 0$ at time $v = 0$ and the final at $v = T$. A central singularity of growing mass developed at $r = 0$. For $v < 0$ we have $m(v) = e(v) = 0$, i.e., the anti-de Sitter metric, and for $v > T$, $\dot{m}(v) = \dot{e}(v) = 0$, $m(v)$ *and* $e^2(v)$ are positive definite. The metric for $v = 0$ to $v = T$ is charged Vaidya-anti-de Sitter, and for $v > T$ we have the Reissner Nordstr$\ddot{o}$m anti-de Sitter solution.

Radial (θ and $\phi = const.$) null geodesics of the metric (1) must satisfy the null condition

$$\frac{dr}{dv} = \frac{1}{2}\left[1 + \alpha^2r^2 - \frac{2m(v)}{r} + \frac{e^2(v)}{r^2}\right]. \tag{7}$$

Clearly, the above differential equation has a singularity at $r = 0$, $v = 0$. The nature (a naked singularity or a black hole) of the collapsing solutions can be characterized by the existence of radial null geodesics coming out from the singularity. The nature of the singularity can be analyzed by techniques in [3]. To proceed further, we choose

$$m(v) = \lambda v \ (\lambda > 0) \ and \ e^2(v) = \mu^2 v^2 (\mu^2 > 0) \tag{8}$$

for $0 \leq v \leq T$ [5, 14]. Let $y \equiv v/r$ be the tangent to a possible outgoing geodesic from the singularity. In order to determine the nature of the limiting value of y at $r = 0$, $v = 0$ on a singular geodesic, we let

$$y_0 = \lim_{r\to 0 \, v\to 0} y = \lim_{r\to 0 \, v\to 0} \frac{v}{r}. \tag{9}$$

Using Eqs. (7), (8) and L'Hôpital's rule we get

$$y_0 = \lim_{r\to 0\, v\to 0} y = \lim_{r\to 0\, v\to 0} \frac{v}{r} = \lim_{r\to 0\, v\to 0} \frac{dv}{dr} = \frac{2}{1 - 2\lambda y_0 + \mu^2 y_0^2} \tag{10}$$

which implies,

$$\mu^2 y_0^3 - 2\lambda y_0^2 + y_0 - 2 = 0. \tag{11}$$

If Eq. (11) admits one or more positive real roots then the central shell focusing singularity is at least locally naked. Thus the occurrence of positive roots implies that CCC is violated. In the absence of positive roots of (11), the central singularity is not naked because in that case there are no outgoing future directed null geodesics from the singularity (for more details, see [3]). Hence when there are no positive roots to (11), the collapse will always lead to a black hole. We now examine the condition for the occurrence of a naked singularity.

An interesting point to note from Eq. (11) is that it admits at least one positive root for $\lambda > 0$ and $\mu^2 > 0$ and no negative roots [12], e.g. Eq. (11) has a positive real $y_0 = 6.26079$ for $\lambda = \mu = 1/4$ and $y_0 = 1.36466$ for $\lambda = 0,\ \mu = 1/2$. It is easy to see that Eq. (11) admits all three positive roots if $\lambda^2 + 18\lambda\mu^2 \geq 16\lambda^3 + \mu^2 + 27\mu^4$. This happens only for $\lambda \leq 0.082$ and $\mu \leq 0.094$. It is then easy to check that positive roots of Eq. (11) $y_0 = 4.53761, 5.74688$ and 9.4686 corresponds to $\lambda = 0.08$ and $\mu = 0.09$. Whereas for $\lambda = 0.04,\ \mu = 0.04$, the roots are $y_0 = 2.46049,\ 16.2218$ and 31.3177. For all such values of y_0, the singularity will be naked. It follows that the gravitational collapse of a charged null fluid in an anti-de Sitter background must lead to a naked singularity regardless of the values of the parameter (λ, μ).

The charged Vaidya metric can be obtained by taking $\alpha = 0$ in Eq. (1), however the Eq. (11) remains unchanged. Thus the results of collapsing shells in anti-de Sitter background are similar to that of collapsing shells of radiation in Minkowskian background [14], as it should have been expected, since when $r \to 0$ the cosmological term $\alpha^2 r^2$ is negligible. The global nakedness of singularity can then be seen by making a junction onto Reissner Nordstr$\ddot{o}$m anti-de Sitter spacetime.

When $\mu = 0$, the metric (1) is Vaidya-anti-de Sitter metric and Eq. (11) admit positive roots when $0 < \lambda \leq 1/16$ and hence singularities are naked for $0 < \lambda \leq 1/16$ [7], which can be shown gravitationally strong [13]. When $\mu = 0$ and $\alpha = 0$ the singularities are naked again for $0 < \lambda \leq 1/16$ in which case the metric is Vaidya metric (see [3], for a review).

The strength of a singularity is an important issue because there have been attempts to relate it to stability [15]. A singularity is termed gravitationally strong or simply strong, if

it destroys by crushing or stretching any object that falls into it. Recently, Nolan [16] gave an alternative approach to check the nature of singularities without having to integrate the geodesic equations. It was shown in [16] that a radial null geodesic which runs into $r = 0$ terminates in a gravitationally weak singularity if and only if $\dot{r}$ is finite in the limit as the singularity is approached (this occurs at $k = 0$), the over-dot here indicates differentiation along the geodesics. So assuming a weak singularity, we have

$$\dot{r} \sim d_0 \quad r \sim d_0 k \tag{12}$$

Using the asymptotic relationship above and Eq. (8), the geodesic equations yield

$$\ddot{v} \sim -(\lambda y_0 d_0^{-1} k^{-1} - \mu^2 y_0^2 d_0^{-1} k^{-1} - \frac{\alpha^2}{3} d_0 k) d_0^2 y_0^2 \tag{13}$$

But this gives

$$\ddot{v} \sim c k^{-1}, \tag{14}$$

, where $c = (\lambda - \mu^2 y_0) y_0 d_0^{-1}$, which is inconsistent with $\dot{v} \sim d_0 y_0$, which is finite. Thus if the coefficient c of k^{-1} is non-zero, the singularity is gravitationally strong. This may be false in the case $c = 0$, which is equivalent to $y_0 = \lambda/\mu^2$. But inserting this into the root Eq. (11) gives

$$\mu^2 = \frac{\lambda}{4}\left(1 \pm (1 - 8\lambda)^{1/2}\right). \tag{15}$$

Thus $c = 0$ corresponds to a set (in fact a closed curve) of measure zero in (λ, μ) parameter space and so is not of physical significance. Therefore, one may say that generically, the naked singularities is gravitationally strong in the sense of Tipler [17].

Having seen that the naked singularities in our model is a strong curvature singularity, we check it for scalar polynomial singularity. The Kretschmann scalar with the help of Eqs. (8), takes the form

$$K = \frac{48}{r^4}\left(\lambda^2 y^2 - 2\lambda\mu^2 y^3 + \frac{7}{6}\mu^4 y^4\right) + 24\alpha^2 \tag{16}$$

which diverges at the naked singularity and hence the singularity is a scalar polynomial singularity.

In this section we discuss gravitational collapse of charged null fluid in plane symmetric and cylindrical symmetric anti-de Sitter spacetimes. Let us first consider the case of plane symmetry. The Einstein-Maxwell equations also have the solution [9]

$$ds^2 = -(\alpha^2 r^2 - \frac{2qm(v)}{r} + \frac{q^2 e^2(v)}{r^2}) dv^2 + 2dvdr + \alpha^2 r^2 (dx^2 + dy^2), \tag{17}$$

where ρ in this case is modified as

$$\rho = \frac{1}{8\pi r^3}\left[qr\dot{m}(v) - \frac{q}{2}e(v)\dot{e}(v)\right]. \tag{18}$$

Here $-\infty < x,\ y < \infty$ are coordinate which describe two dimensional zero curvature space which has topology $R \times R$. The parameter q has value $2\pi/\alpha^2$, is taken from Arnowitt-Deser-Misner (ADM) mass of corresponding static black hole found in [9]. The metric (17) is plane symmetric Vaidya-like metric representing gravitational collapse of charge null dust in plane symmetric anti-de Sitter spacetime. Setting $m(v) = const$, $e(v) = 0$ and $\alpha = 0$ one obtains the Taub metric [18]. As in the spherically symmetric case, the physical situation is that of a radial influx of charged null fluid towards the centre. The first ray arrives at the centre for $r = 0$, $v = 0$, and final shell arrives at $v = T$, say, then can be matched with exterior static spacetime. For $v < 0$ we have plane symmetric anti-de Sitter spacetime.

Since the Kretschmann scalar is given by

$$K = \frac{48q^2}{r^6}\left[m^2(v) - \frac{2q}{r}e^2(v)m(v) + \frac{7q^2}{6}\frac{e^4(v)}{r^2}\right] + 24\alpha^2 \tag{19}$$

, there is scalar polynomial singularity at $r = 0$ for m and $e \neq 0$. As above, further analysis of structure of this singularity is initiated by study of the radial null geodesics equation

$$\frac{dr}{dv} = \frac{1}{2}\left[\alpha^2 r^2 - \frac{2qm(v)}{r} + \frac{q^2e^2(v)}{r^2}\right]. \tag{20}$$

Again Eq. (20) has a singular point at $r = 0$ and $v = 0$ and we write as in Eq. (8)

$$qm(v) = \lambda v\ (\lambda > 0)\ and\ q^2e^2(v) = \mu^2 v^2 (\mu^2 > 0) \tag{21}$$

and for this case the algebraic equation is

$$\mu^2 y_0^3 - 2\lambda y_0^2 - 2 = 0. \tag{22}$$

Eq. (22) has atleast one positive roots and no negative roots (for e.g. a root $y_0 = 2.3593$ of Eq. (22) corresponds to $\lambda = \mu = 1$). For the present case Eq. (13) is unaltered and hence singularities are strong curvature singularities. Thus referring to above discussion, collapse lead to a naked singularity irrespective of the values of the parameter. The uncharged case can be obtained by taking $\mu = 0$ and Eq. (22) does not admit any positive roots and hence collapse proceed to form a black hole [7].

Finally, we turn our attention to the cylindrical symmetric ant-de Sitter spacetime. The metric is

$$ds^2 = -(\alpha^2 r^2 - \frac{2qm(v)}{r} + \frac{q^2 e^2(v)}{r^2})dv^2 + 2dvdr + r^2(d\theta^2 + \alpha^2 dz^2), \tag{23}$$

where $-\infty < v,\ z < \infty$, $0 \leq r < \infty$ and $0 \leq \theta \leq 2\pi$. Here ρ is given by Eq. (18), but parameter q has value $2/\alpha$. The two dimensional surface has topology of $R \times S^1$. Similar to the above discussion is also valid in this case as well. Therefore, we can conclude that the gravitational collapse in the cylindrical symmetric anti-de Sitter spacetime also forms a strong curvature naked singularities which are also scalar polynomial.

In this work we have discussed the spherical and the non-spherical (planar and cylindrical) collapse of charged null fluid in an anti-de Sitter background. In the limit $\mu \to 0$ our results reduce to those obtained previously [7]. For the spherical case, the collapse proceeds in much the same way as in Minkowskian background [14] and as in the uncharged case [7], i.e., shell focusing strong curvature naked singularities do arise violating CCC, but not the hoop conjecture. Lemos [7] has shown that non-spherical null fluid (uncharged) collapse does not yield naked singularities, but always black holes. We have shown the inclusion of charge does alter the result, i.e., non-spherical collapse of charged null fluid leads to strong curvature singularities. This shows that in our non-spherical case also the CCC is violated, but not the hoop conjecture.

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